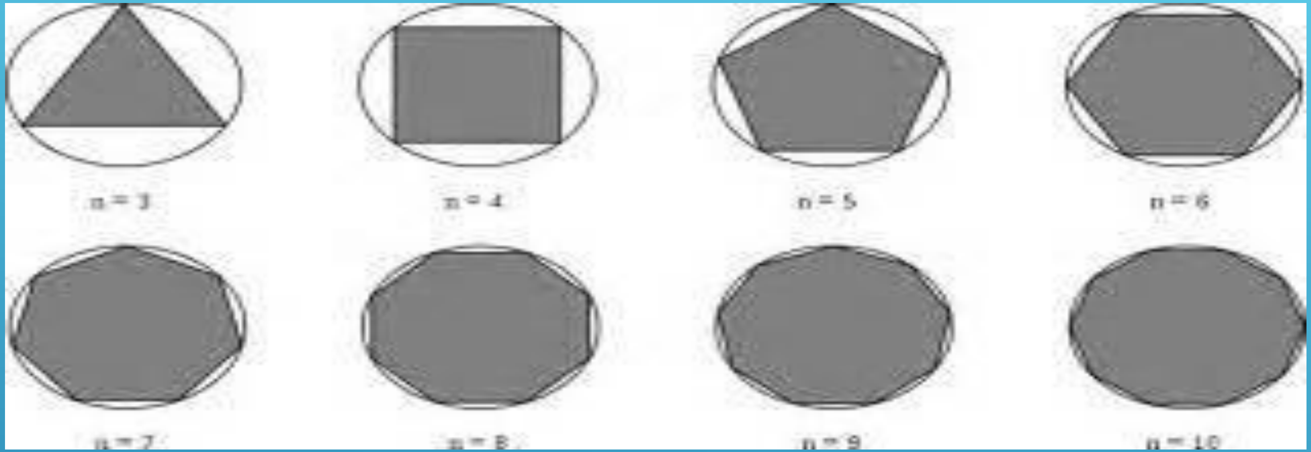
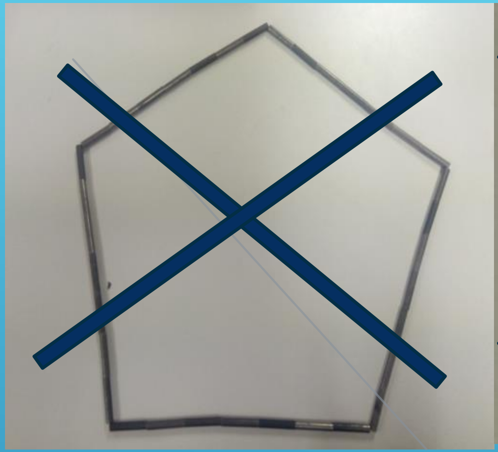
CIRCLE MAGNETS

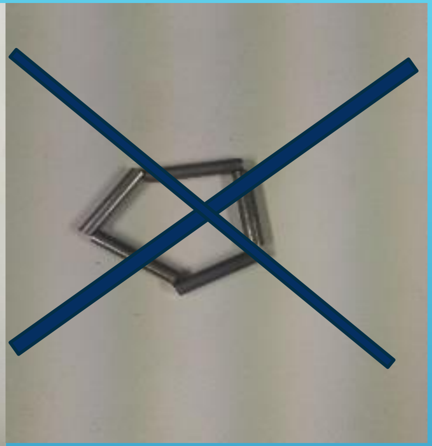
"If you stack many small cylindrical magnets, the resulting stick of the magnets will have some elasticity. Is it possible make it sufficiently elastic to join both ends of the magnet stick? If yes, what is the minimal attainable **ratio of the radius of the resulting circle of magnets to the single magnet radius**?"

Vilage Muriel ,
L3 de Physique
JUFR 2019

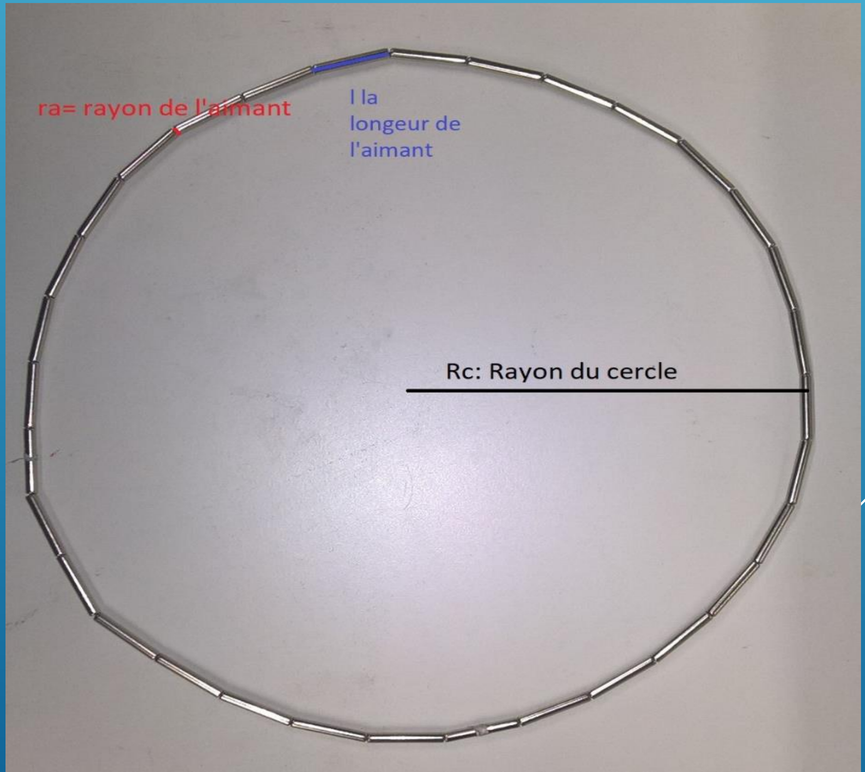
The approximation of a regular polygon by a circle



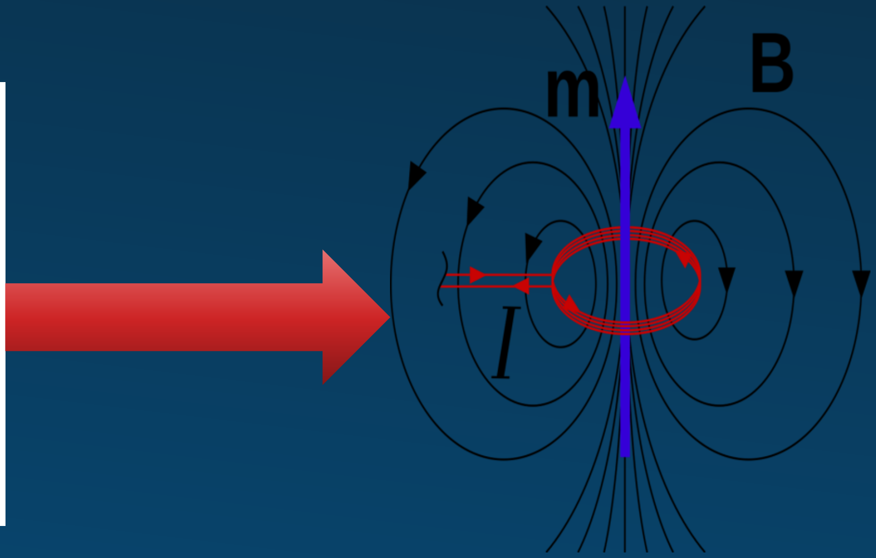




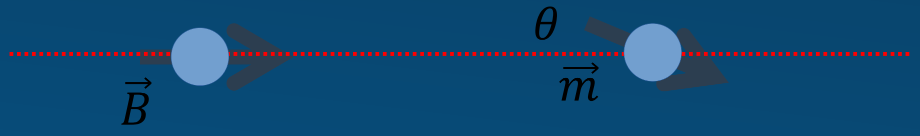
$$\frac{R_c}{r_a} = F\left(\frac{l}{ra}\right)?$$



THEORETICAL MODEL



$$\vec{B}(r) = \frac{\mu M}{2 * r^3}\overrightarrow{Uz}$$



$$E_P = -\vec{m} * \vec{B}$$

- Dipolar Approximation
- Potential energy between two dipoles
- Elasto-Plastic transition between two configurations: break and curved configuration

1) Curved Configuration

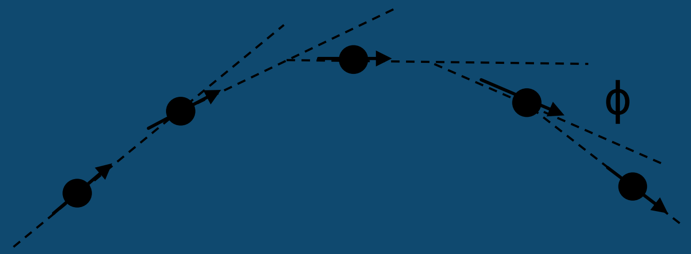
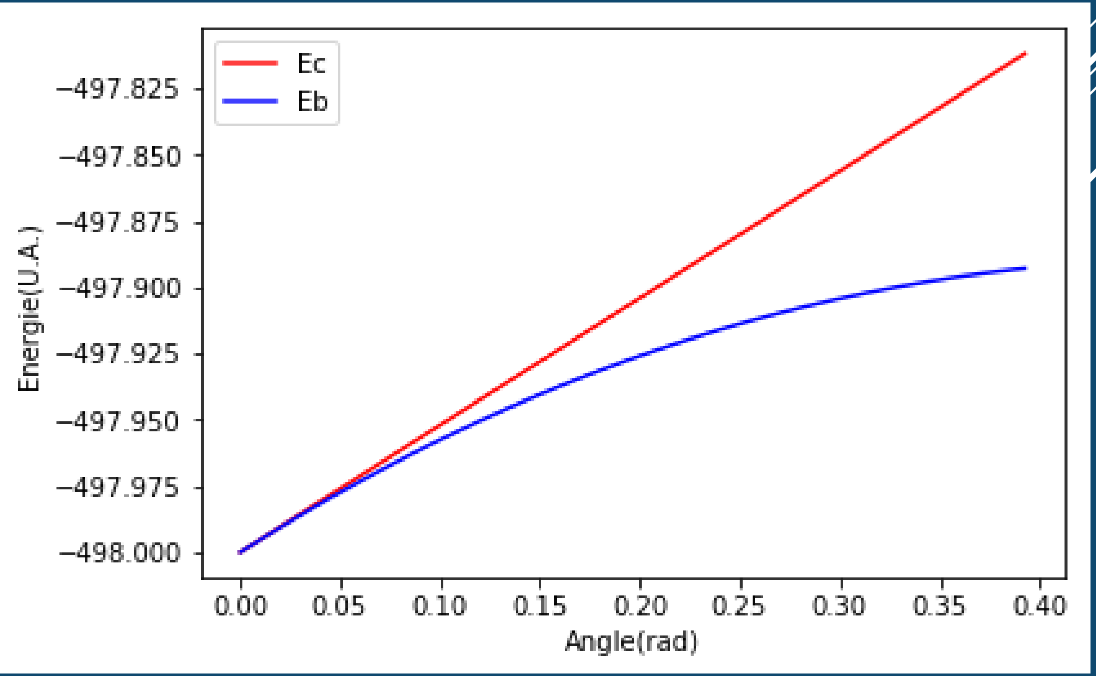


Figure 1: Chain of magnets with uniform repartition of angle ϕ

L:lengh of the magnet
a:diameter of the magnet
N: number of magnets
m: magnetic moment
B: magnetic field
φ: angle between two successive magnets



2) Break configuration

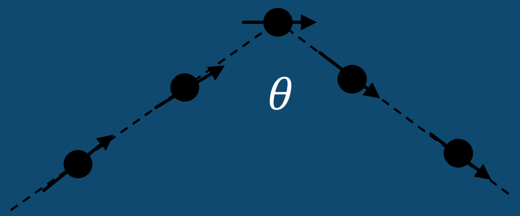
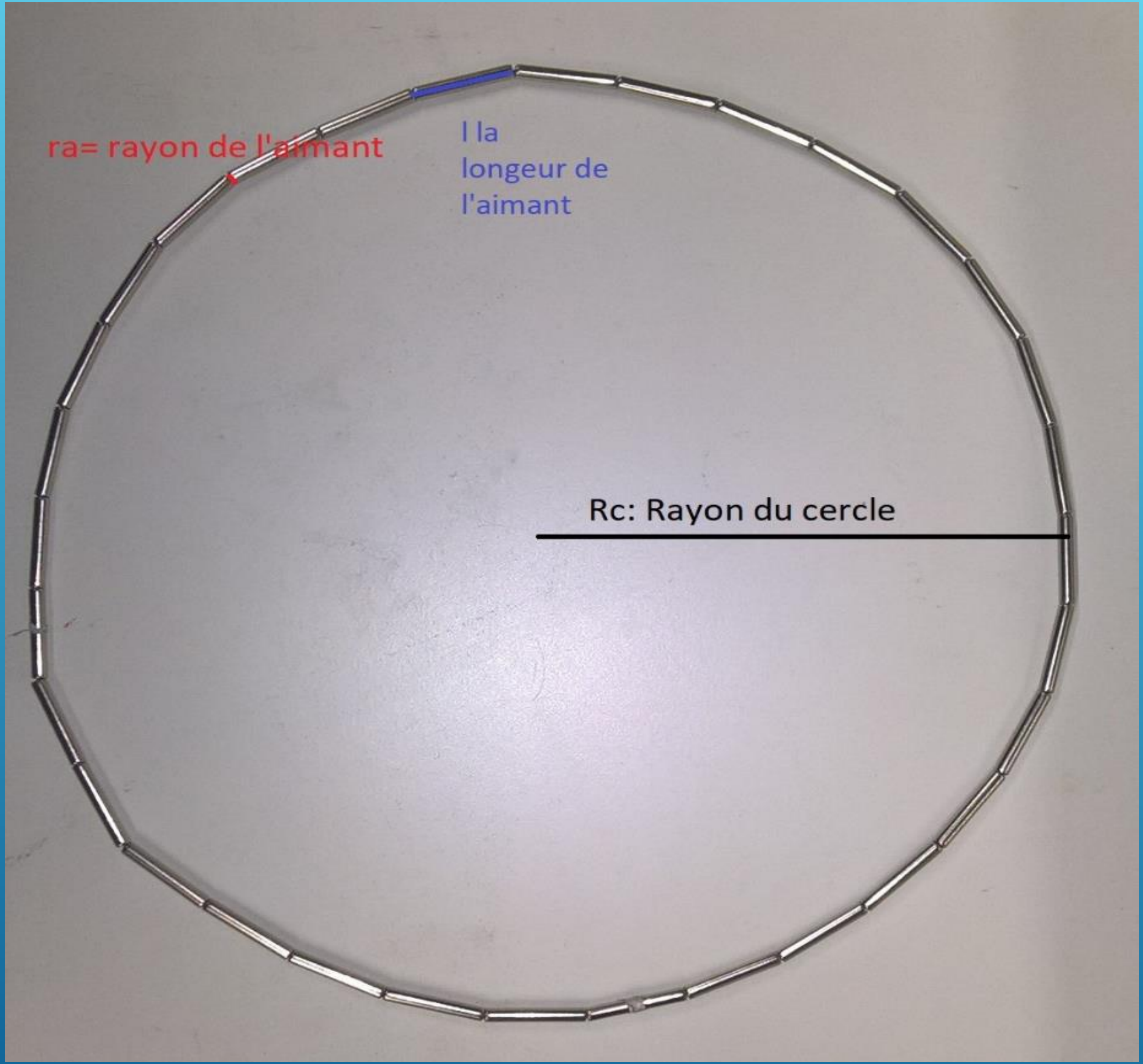


Figure 2: Localisation of the stress

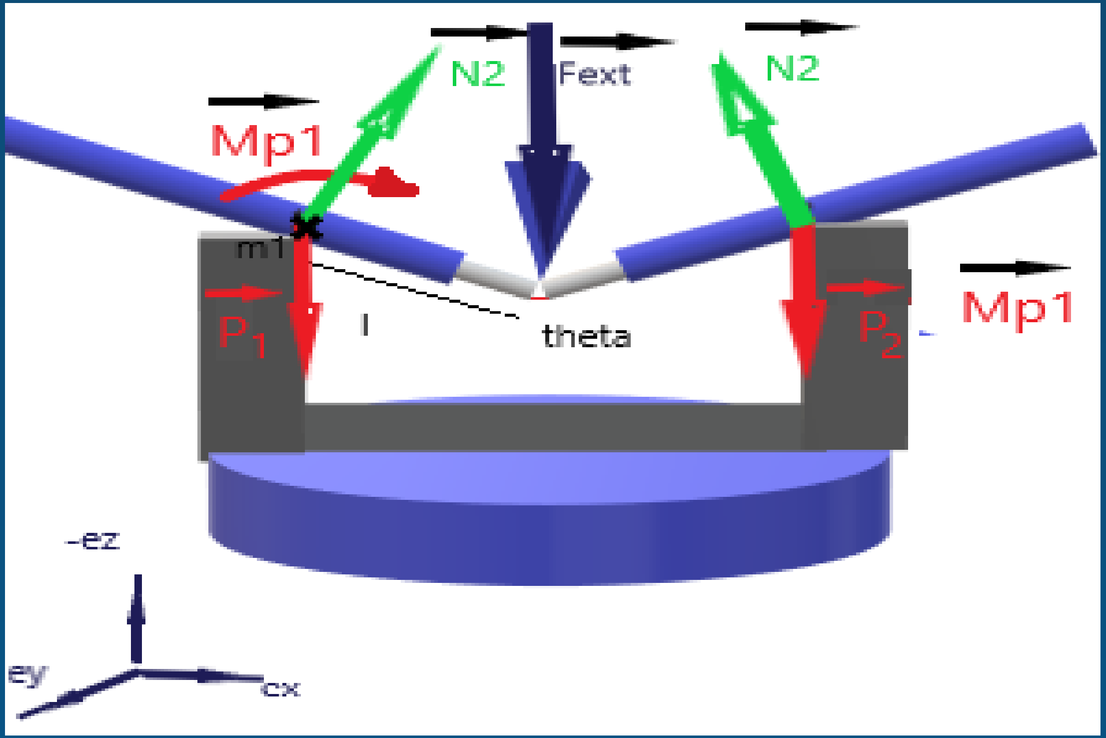

ra= rayon de l'aimant
l la
longeur de
l'aimant
Rc: Rayon du cercle

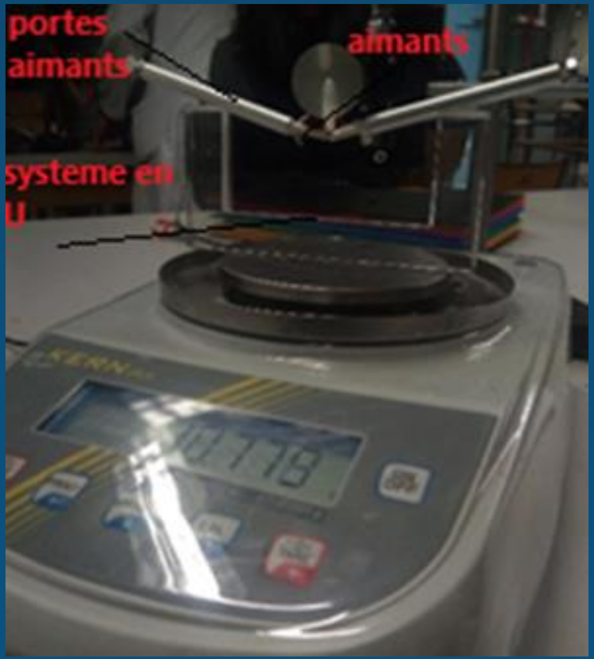
Elasticity: the magnetic force Moment increase with the angle

The circle magnet is in stable equilibrum if and only if $\frac{d^2E}{d\theta^2}$ >0 (E the potential energy between two neighbours magnets torn apart from an angle θ) ie $\frac{dM}{d\theta}$ >0 (M the Magnetic Force Moment)

- Theorically the energy curve is always concave, however in reality we observe circle magnets, why?

EXPERIMENTAL DETERMINATION OF THE MAGNETIC FORCE MOMENT





Thanks to the fundamental principle of dynamics we can show that the magnetic force moment is linked to the mass read on the weighter as below:

$$\overrightarrow{M_{mag}} = g \times m_{lu} \times l \ \overrightarrow{ey}$$

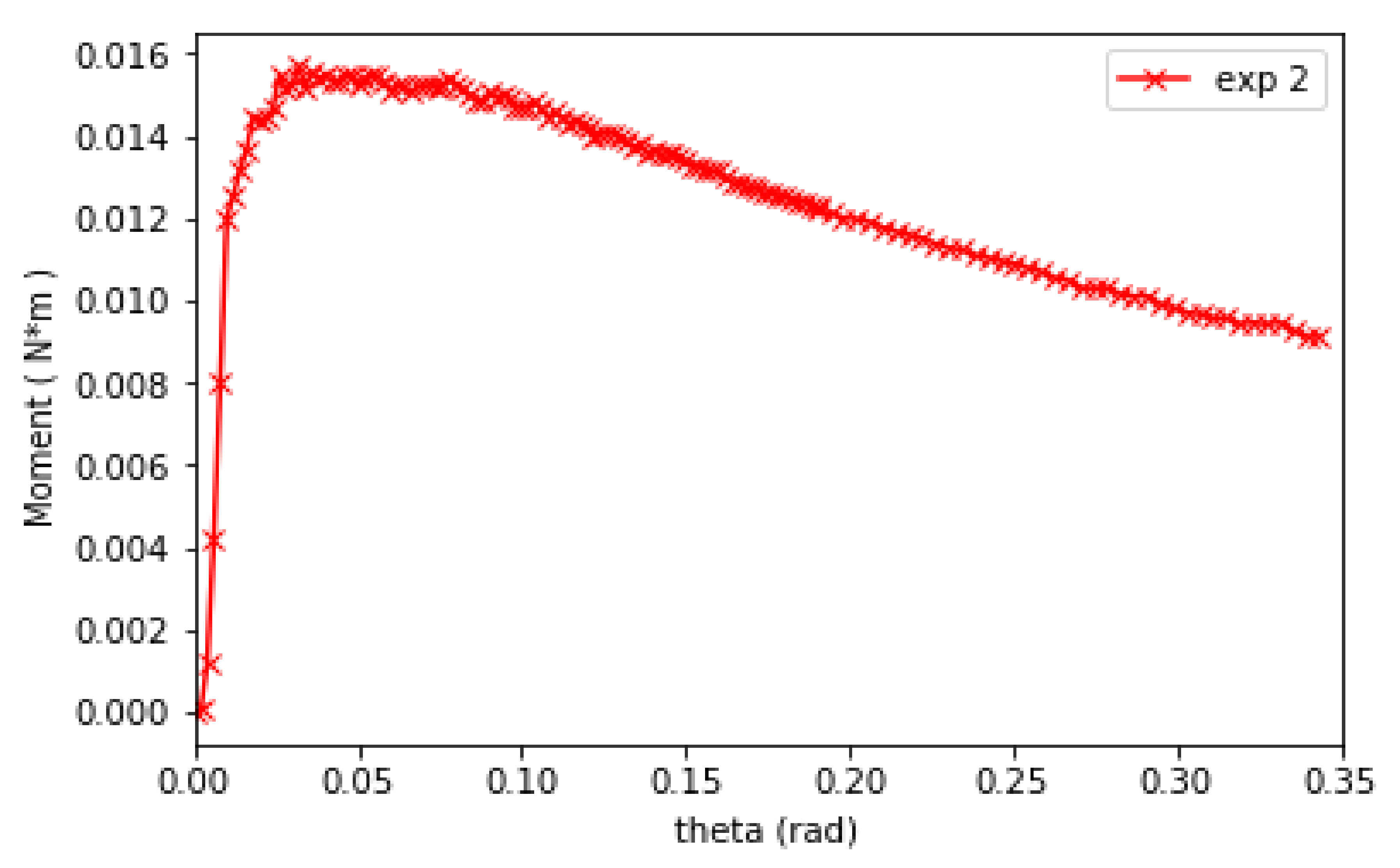
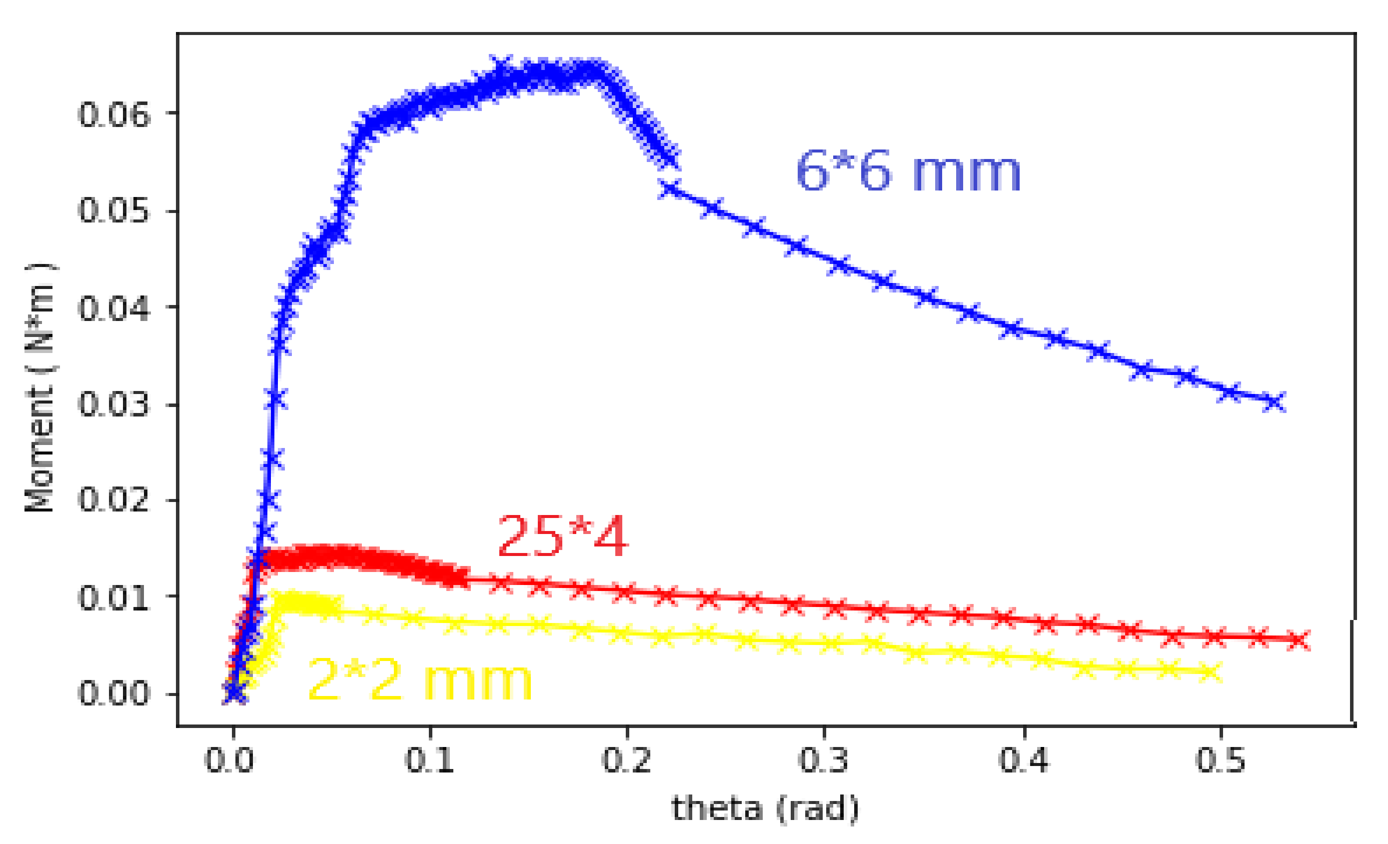


Figure 2: evolution of the Magnetic force moment in fonction of the angle between two magnets (25*4 mm NdFb magnets)

$$\theta = \frac{2\pi}{N}$$

Variation of the radius of the magnet



Similar curvature however we need more strenght to torn appart magnets with highter radius

The range of angles for a stable circle configuration seems to increase with the radius of the magnet

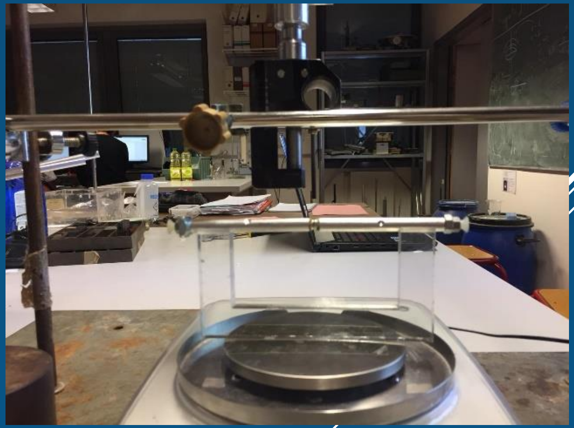
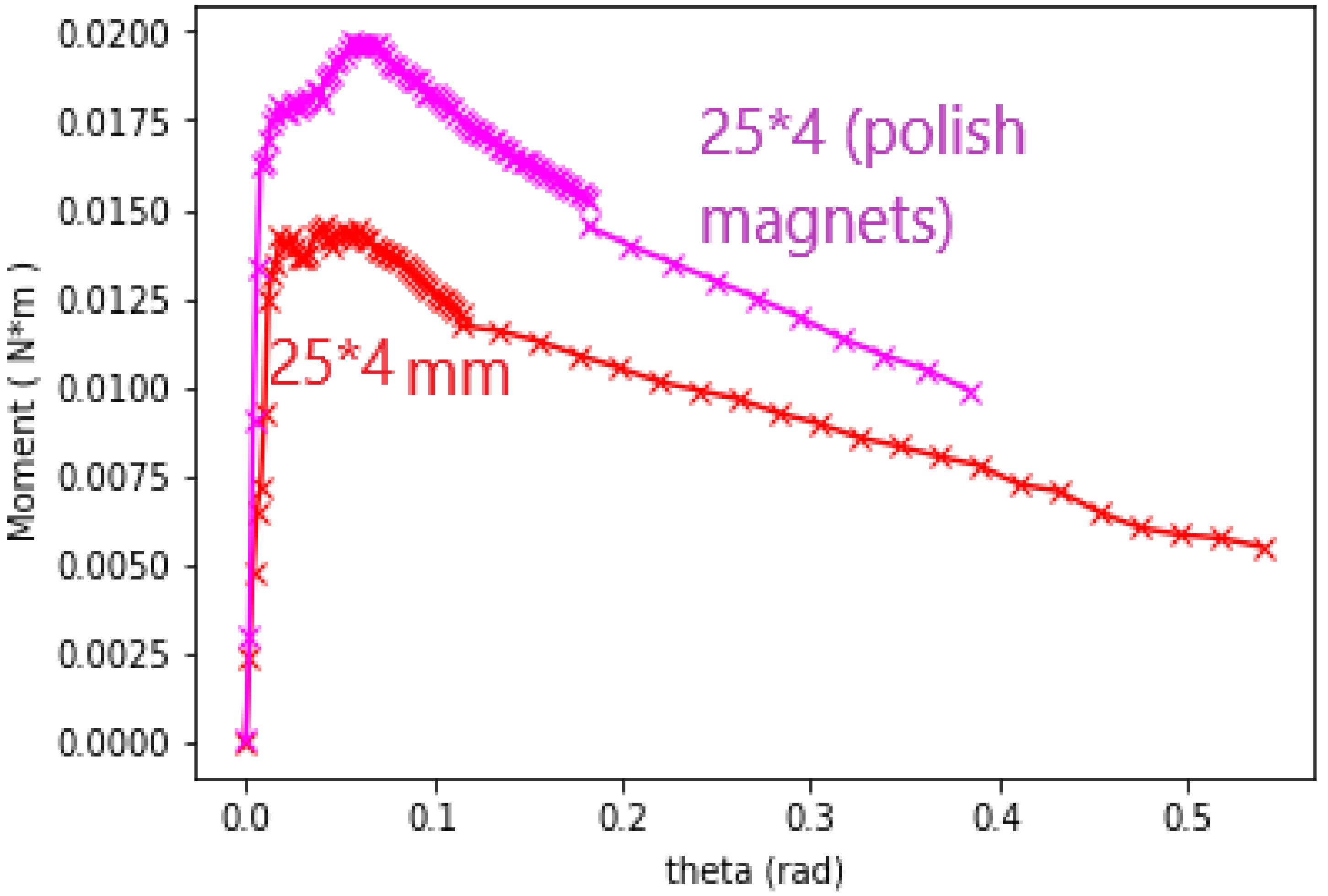


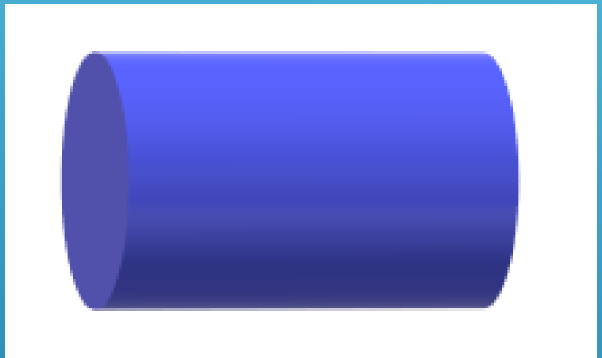
Figure 2: Magnetic Force Moment : 2*2 mm (=l*2*ra) (yellow) 25*4 mm (red) ,6*6 mm(blue)

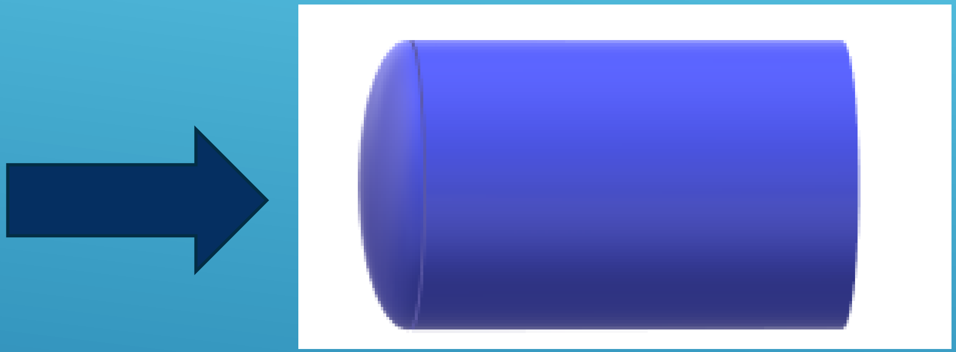
Rugosity



Rought Magnets

Cylindro- spherical magnets





Perfect cylindrical magnet

Non flat real magnet

CONCLUSION

-Elasticity? In our theory,no. In reality yes, mainly because the magnets are not perfectly shaped.
-Experimentaly the circle size decrease with an increasing radius

To go Further:

Other effects that stabilized our circle:

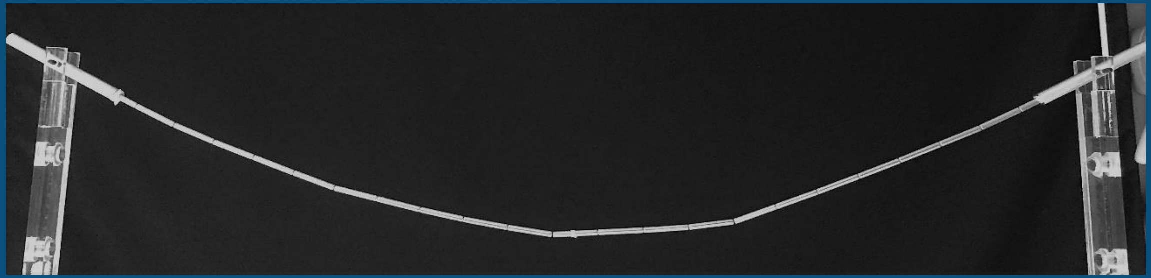
- Study the friction(with the table), gravity

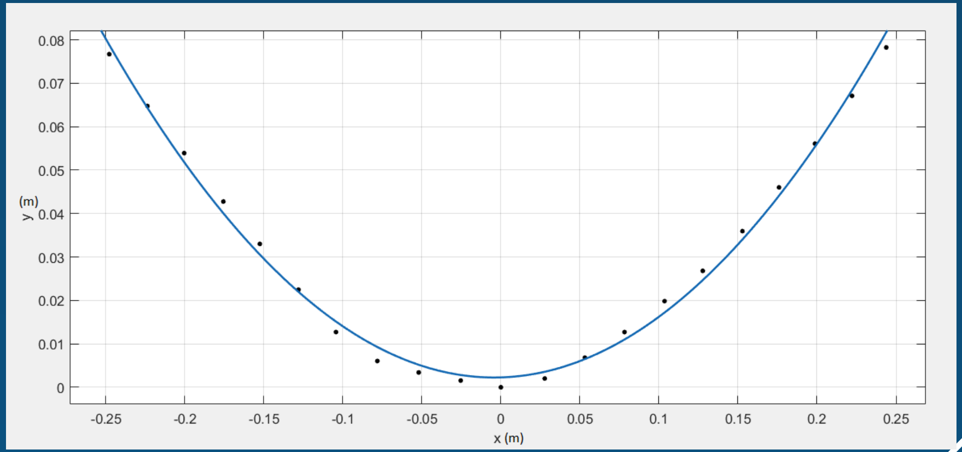
-measure the rugosity

- cylindro spherical transition
- Rigidity of the magnet
- -Elasticity of the material itself

Forçage du cercle

MANIPULATION SOUS GRAVITÉ





Transition élasto-plastique?

ANNEXES SLIDE 9: DÉMONSTRATION : $\overrightarrow{M_{mag}} = g \times m_{lu} \times l\ \overrightarrow{ey}$

Nous pouvons le démontrer grâce au principe fondamental de la dynamique

- <u>Systèmes</u>: $-1.\{aimant\ 1|tige\ 1\}\quad -2.\{2\ aimants\ |2\ tiges\}$
- PFD en **statique** sur le *système 1* :
- $\overrightarrow{N'_2} + \overrightarrow{N_1} + \overrightarrow{Fmag} + \overrightarrow{P_1} + \overrightarrow{F_{ext}} = \vec{0}$
- Sur le *système* 2 :
- $\overrightarrow{N_1} + \overrightarrow{N_2} + \overrightarrow{P_1} + \overrightarrow{P_2} + \overrightarrow{F_{ext}} = \vec{0}$
- $/\overrightarrow{ez}$: $--N_{1z} - N_{2z} + P_1 + P_2 + F_{ext} = 0$
- On tare la balance pour initialiser à 0 le poids du système : {aimants}U{porte-aimant}U{système en U} donc $P_1 + P_2$ est taré au début de l'expérience ie dans l'équation ci-dessus P1+P2=0
- De plus $m_{lu} = (N_{1z}+N_{2z})/g$ (avec g constante de gravitation terrestre) d'où $F_{ext} = m_{lu} \times g = (N_{1z}+N_{2z})$
- Théorème des Moments **au point m1** : $\overrightarrow{M_{ext}} + \overrightarrow{M_{mag}} = \vec{0}$
- Or **au point m1** $\overrightarrow{M_{ext}} = -|\overrightarrow{F_{ext}}| \times |Om1|\overrightarrow{ey}$ on pose l = Om1 $\overrightarrow{M_{ext}}$=- g×$m_{lu} \times l\overrightarrow{ey}$
- $ie\ \overrightarrow{M_{ext}} = -|\overrightarrow{N_1} + \overrightarrow{N_2}| \times |Om1|\overrightarrow{ey} = -g \times m_{lu} \times l\ \overrightarrow{ey}$
- puis $\overrightarrow{M_{mag}} = g \times m_{lu} \times l\ \overrightarrow{ey}$

Annexes slide 7 Reproductibilité et répétabilité de l'experience

Ordres de Grandeurs des appareils de mesures et Incertitudes

Incertitudes:

-teslamètre (constructeur, dernier digit) est de 0.1 mT
constructeur balance 1: 0.001 g
balance 2: 0.01g ;
de lecture vernier 0.01mm

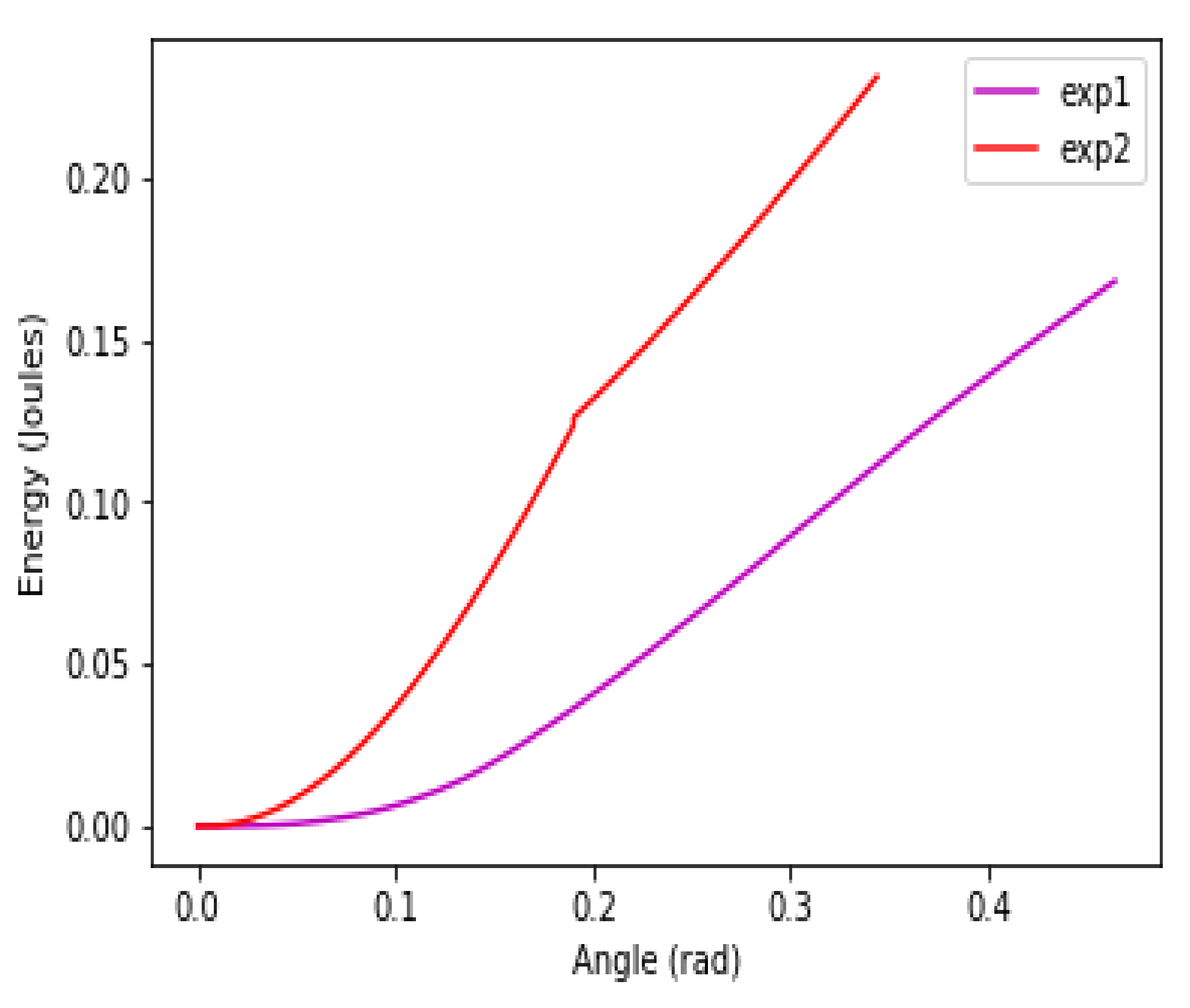
Domaine de mesure des appareils:

-Teslametre [0,100] mT
-Balance [0,1200] grammes

Annexe python

```
##############expérience 2, Néodyme 10*25 mm  aimant ne bouge pas dans les tubes########

o=[0,0.07,2.35,8.6,16.3,24.5,25.6,26.8,27.9,29.4,29.3,29.5,29.9,31.6,30.9,31.1,32,30.9,31.7
   ,31.4,31.5,31.3,31.2,31.6,31.6,31.0,31.4,31.5,31.5,31.2,30.8,31.1,31,30.7,31.0,31,31.1,30
   31.1,31.4,31.0,30.9,30.6,30.3,30.2,30.7,30.5,30.6,30.5,30,30,30,30.2,29.9,29.5,29.7,29.5,
   29,28.5,28.7,28.7,28.7,28.5,28.3,28,28.1,27.7,27.8,27.6,27.7,27.6,27.5,27.3,27.1,27,26.9,
   26.2,26.2,26,26,25.9,25.7,25.7,25.6,25.5,25.4,25.3,25.3,25.1,25]  #poids balance en gram

k=[24.9,24.7,24.5,24.4,24.28,24,23.8,23.6,23.5,23.2,23,22.9,22.6,22.5,22.3,22.2,22,21.8,21.5
   21,21,20.7,20.6,20.6,20.2,20,19.8,19.7,19.5,19.5,19.2,19.3,19.2,19.3,18.9,18.6,18.6] #poi

y=o+k #2 lists car on a pris deux intervalle de z différant v1 et v2
M2=np.zeros(len(y)) # force
Mo2=np.zeros(len(y)) #moment du force
E2=np.zeros(len(y))   #energie

#v1 jusqua v5 juste changement de list en np.array pour pouvoir appliquer des opérateur faci

v1=np.arange(0,96*0.0001*0.5,0.0001*0.5)
v2=np.arange(0.00475,0.00475+39*0.0001,0.0001)
v3=v1.tolist()
v4=v2.tolist()
v5=v3+v4
z2=np.asarray(v5)

theta2=2*np.arcsin(z2/l)
dt2=theta2[50]-theta2[49]

for k in range(len(y)):
    M2[k] = y[k]*10**-3

for k in range(len(y)):
    Mo2[k] = y[k]*10**-3*g * l #10**-3 gram à kilo

for i in range(0,len(y)-1):
    E2[i+1]=E2[i]+Mo2[i]*dt2
```

Intégration par la méthode des rectangles et obtention des courbes d’énérgies: Concave ou convexe?



Considérons plus généralement que certains angles initialement ouvert d'un angle θ vont se déformer en prenant un angle $\theta + \varepsilon_i$, $\varepsilon_i \in [\varepsilon_1, \varepsilon_n]$

On notera U l'énergie potentielle entre deux aimants voisins qui sont écartés d'un angle θ

Le cercle est une configuration plus stable que le polygone déformé si et seulement si U est localement convexe. En effet l'énergie totale de la configuration déformée est équivalente pour $\varepsilon_i \ll \theta$ à :

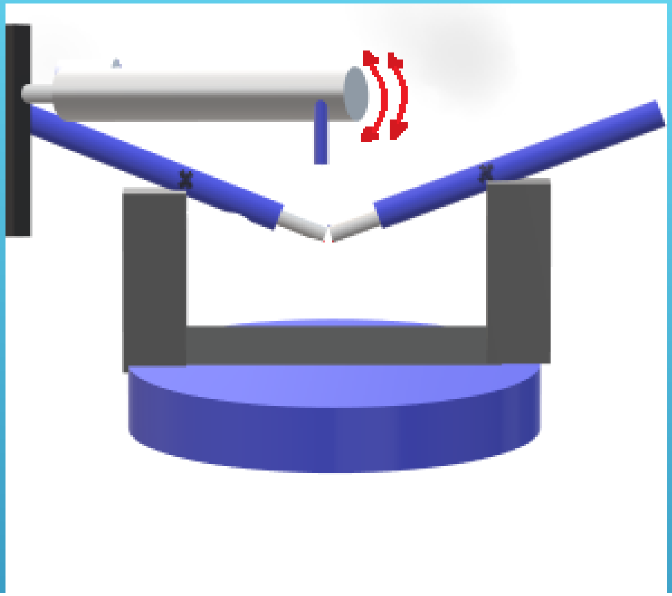
$$\sum \mathrm{U}(\theta + \varepsilon_i) = \sum \mathrm{U}(\theta) + \sum \varepsilon_i \times \partial \mathrm{U}(\theta)/\partial \varepsilon_i + \sum \frac{{\varepsilon_i}^2}{2} \times \partial^2 \mathrm{U}(\theta)/\partial^2 \varepsilon_i + \ldots$$

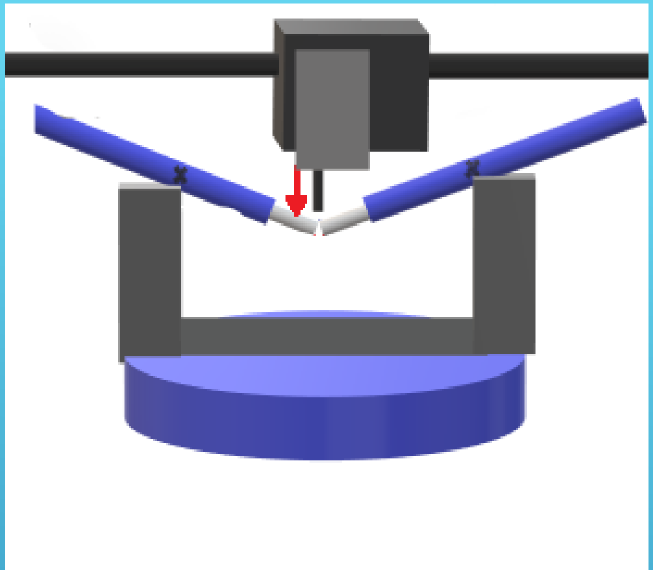
(Application de la formule de Taylor Young)

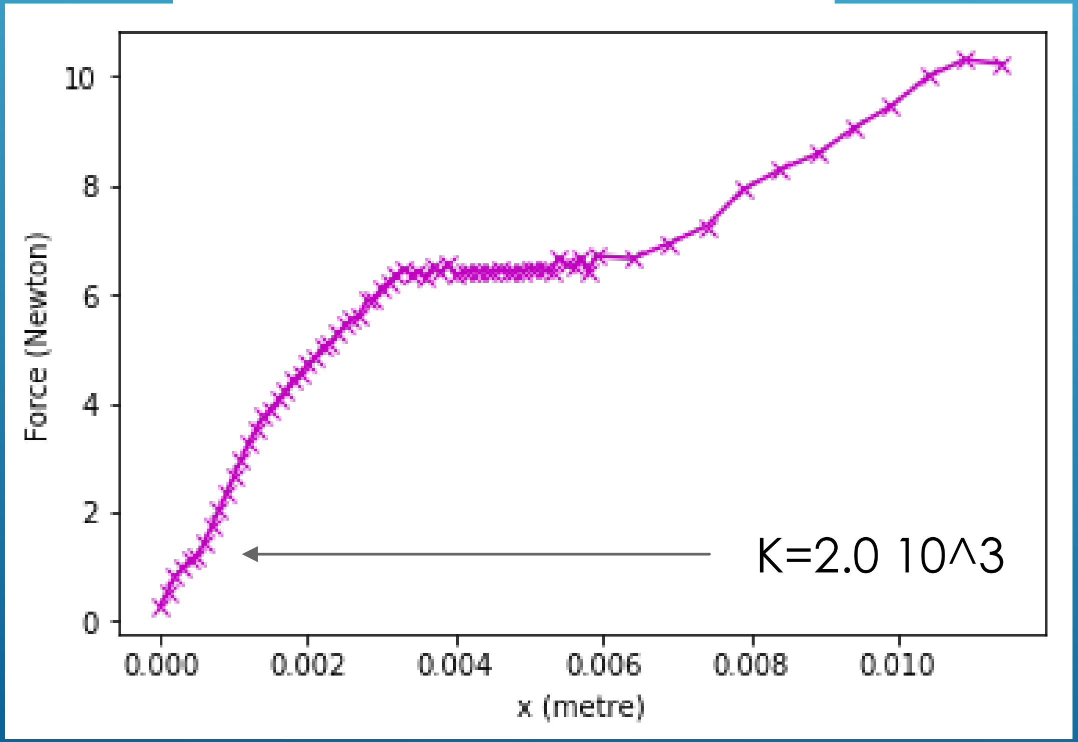
Avec $\mathrm{U}(\theta + \varepsilon_i) = \mathrm{U}(\theta) + \varepsilon_i \times \partial \mathrm{U}(\theta)/\partial \varepsilon_i + \frac{{\varepsilon_i}^2}{2} \times \partial^2 \mathrm{U}(\theta)/\partial^2 \varepsilon_i$

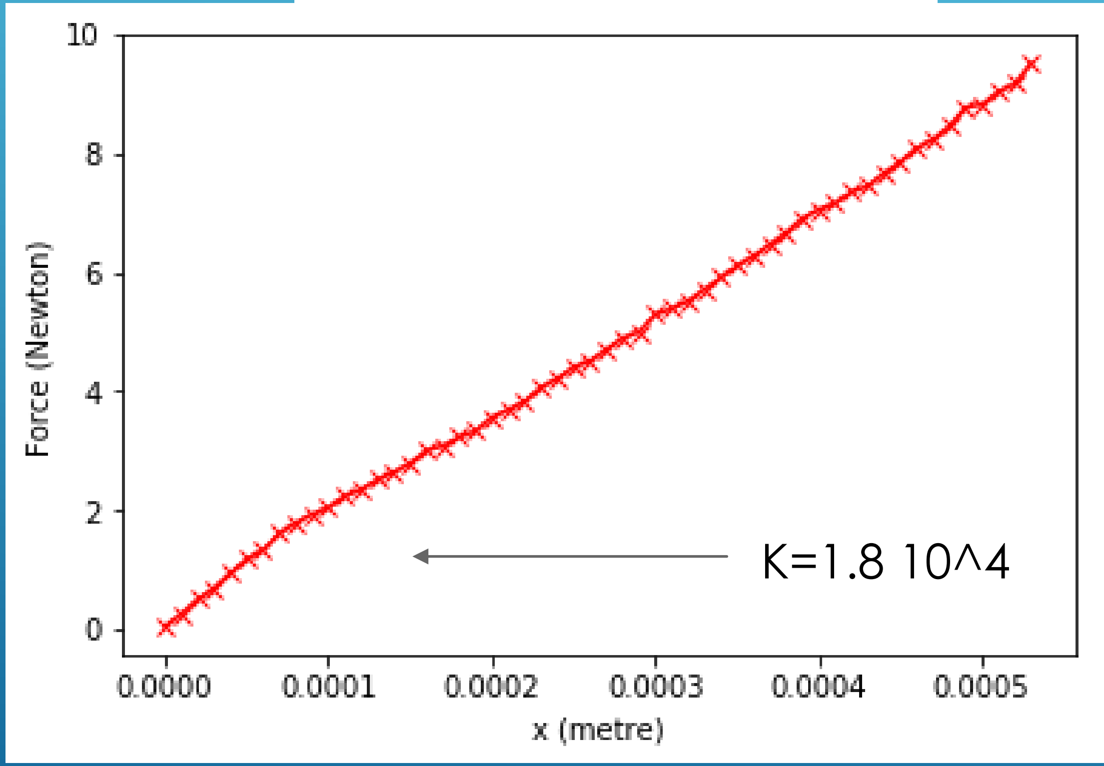
Notons Uo = $\sum \mathrm{U}(\theta)$ l'énergie du cercle :

$$\sum \mathrm{U}(\theta + \varepsilon_i) = \mathrm{Uo} + \sum \varepsilon_i \times \partial \mathrm{U}(\theta)/\partial \varepsilon_i + \sum \frac{{\varepsilon_i}^2}{2} \times \partial^2 \mathrm{U}(\theta)/\partial^2 \varepsilon_i$$

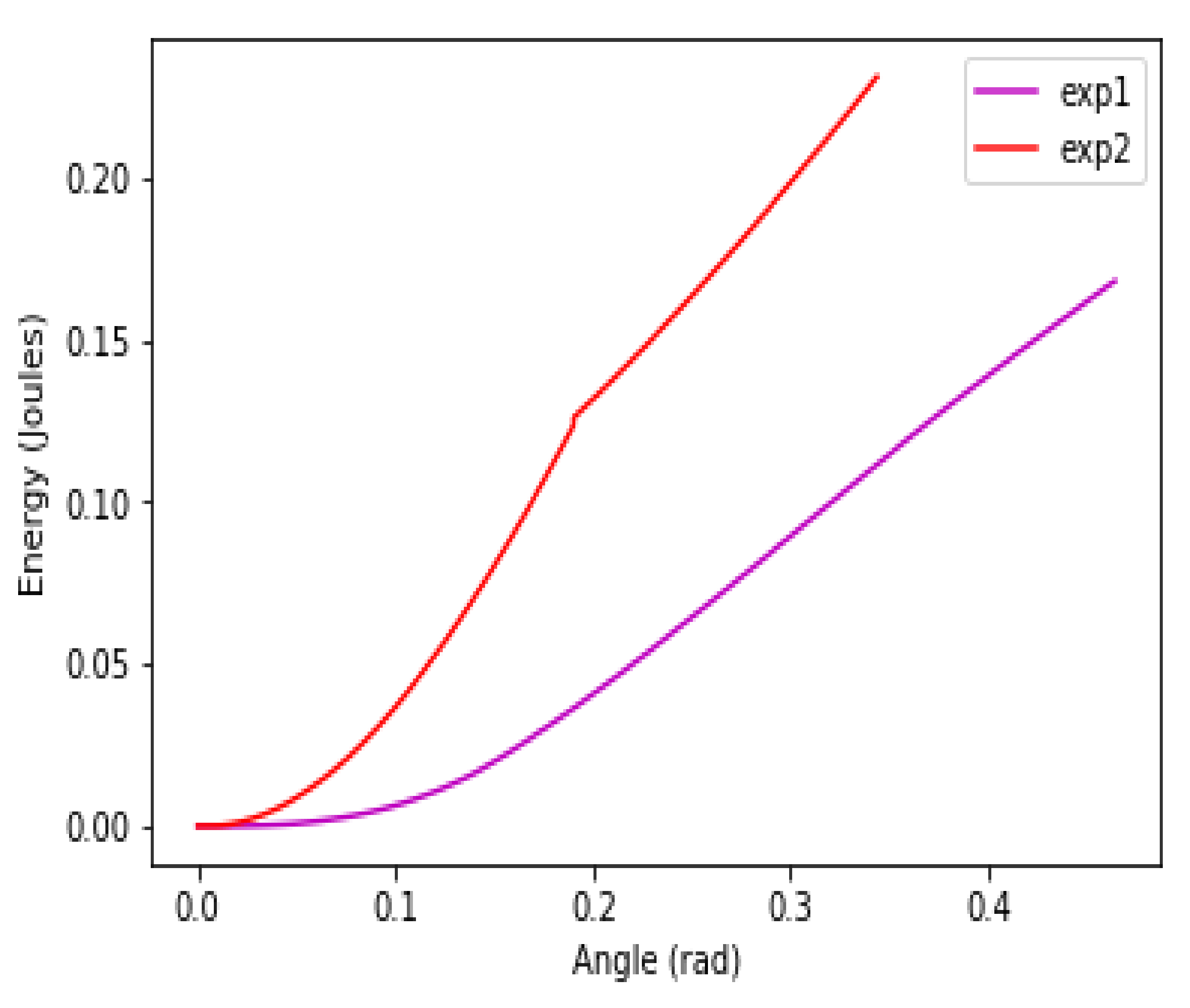


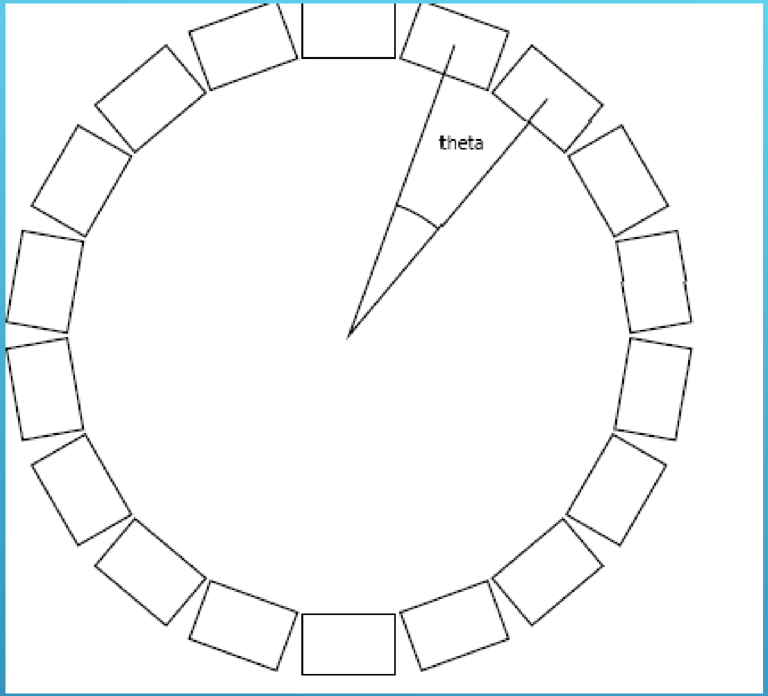


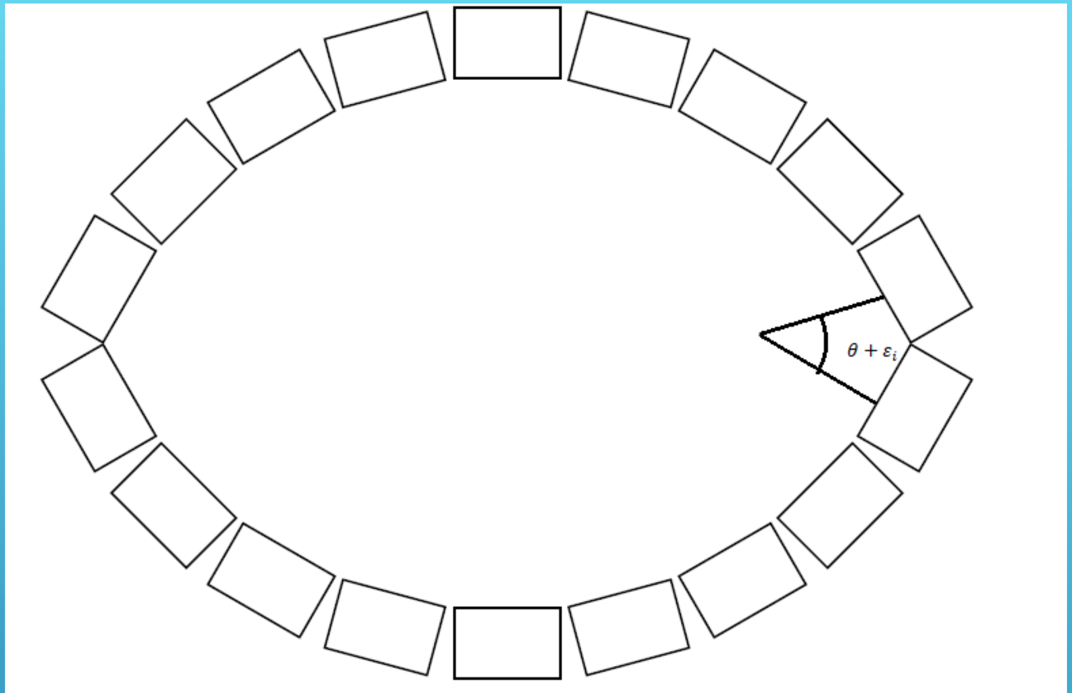

Force (Newton)
0
2
4
6
8
10
0.000
0.002
0.004
0.006
0.008
0.010
x (metre)
K=2.0 10^3

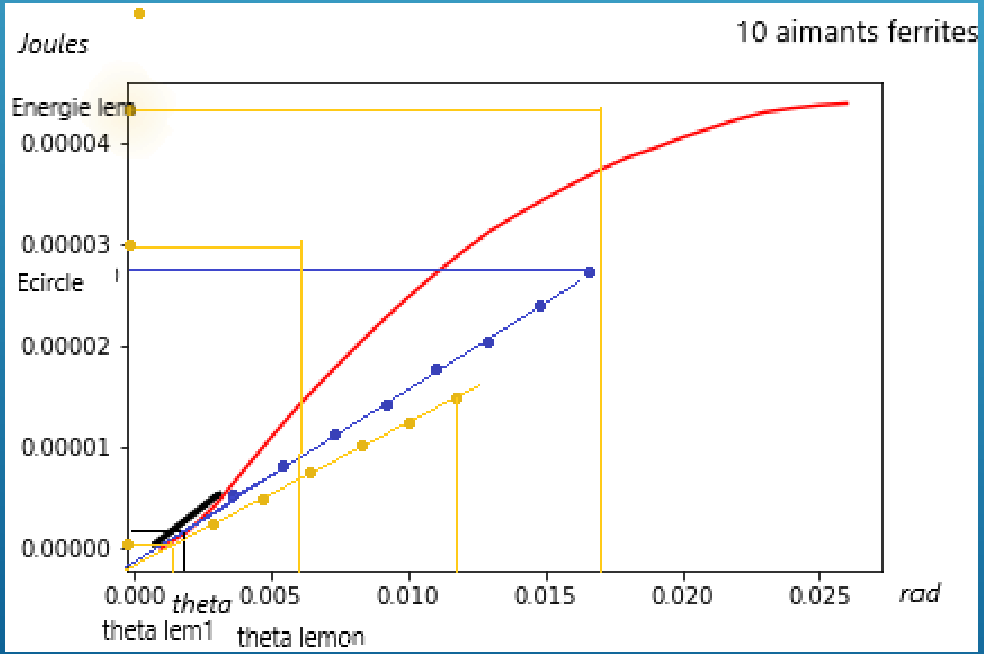

Force (Newton)
0
2
4
6
8
10
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
x (metre)
K=1.8 10^4

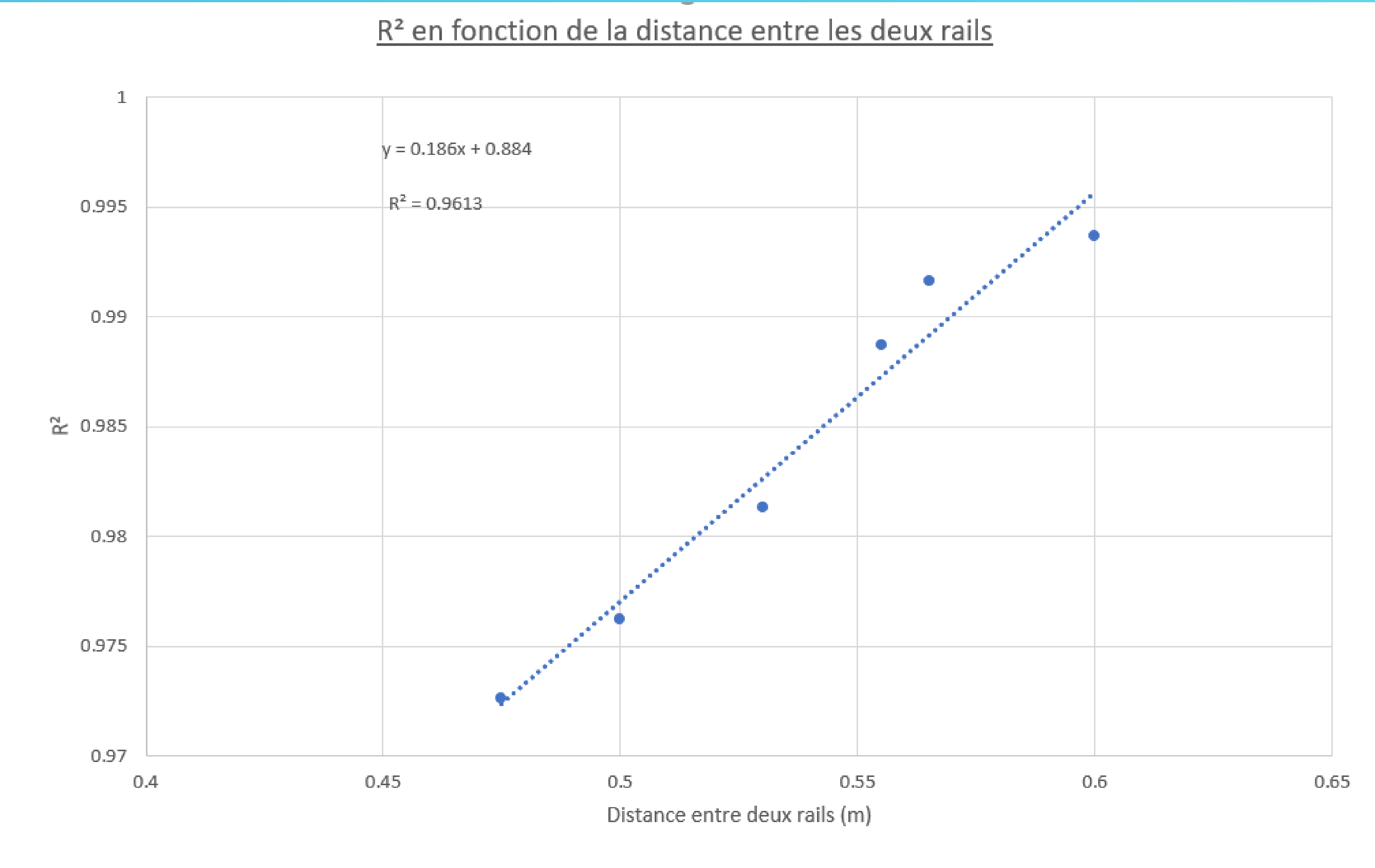
Intégration par la méthode des rectangles et obtention des courbes d'énérgies: Concave ou convexe?



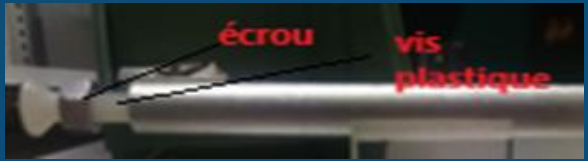

theta


$\theta + \varepsilon_i$


Joules
10 aimants ferrites
Energie lem
0.00004
0.00003
Ecircle
0.00002
0.00001
0.00000
0.000
theta
0.005
0.010
0.015
0.020
0.025
rad
theta lem1
theta lemon


R² en fonction de la distance entre les deux rails
y = 0.186x + 0.884
R² = 0.9613
R²
1
0.995
0.99
0.985
0.98
0.975
0.97
0.4
0.45
0.5
0.55
0.6
0.65
Distance entre deux rails (m)

LES SOURCES D'ERREURS EXPÉRIMENTALES

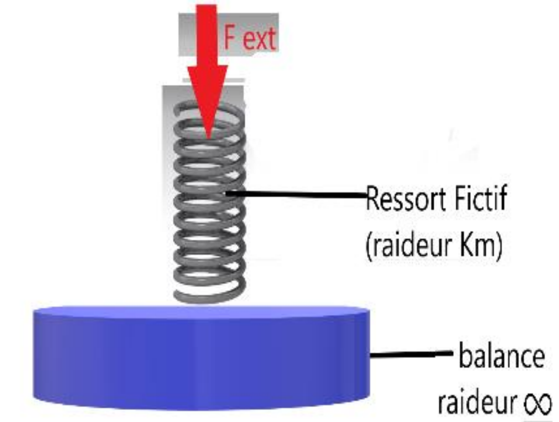


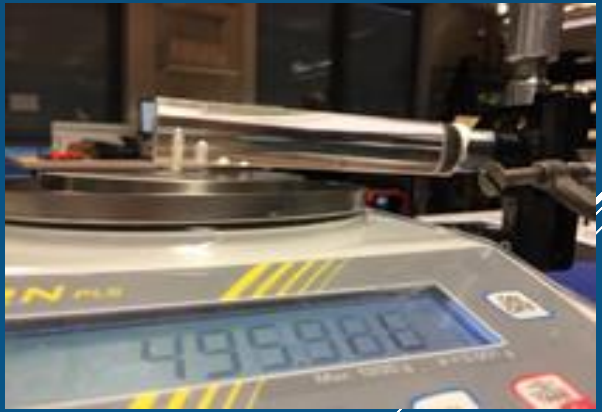
placer les tiges aux centres de gravité (idéal) de chacune d'elles sur le U

Fixer l'aimant avec du téflon dans le porte aimant

Incertitudes : 1- constructeur balance 1: 0.001 g ; balance 2 0.01g
2- lecture vernier 0.01mm

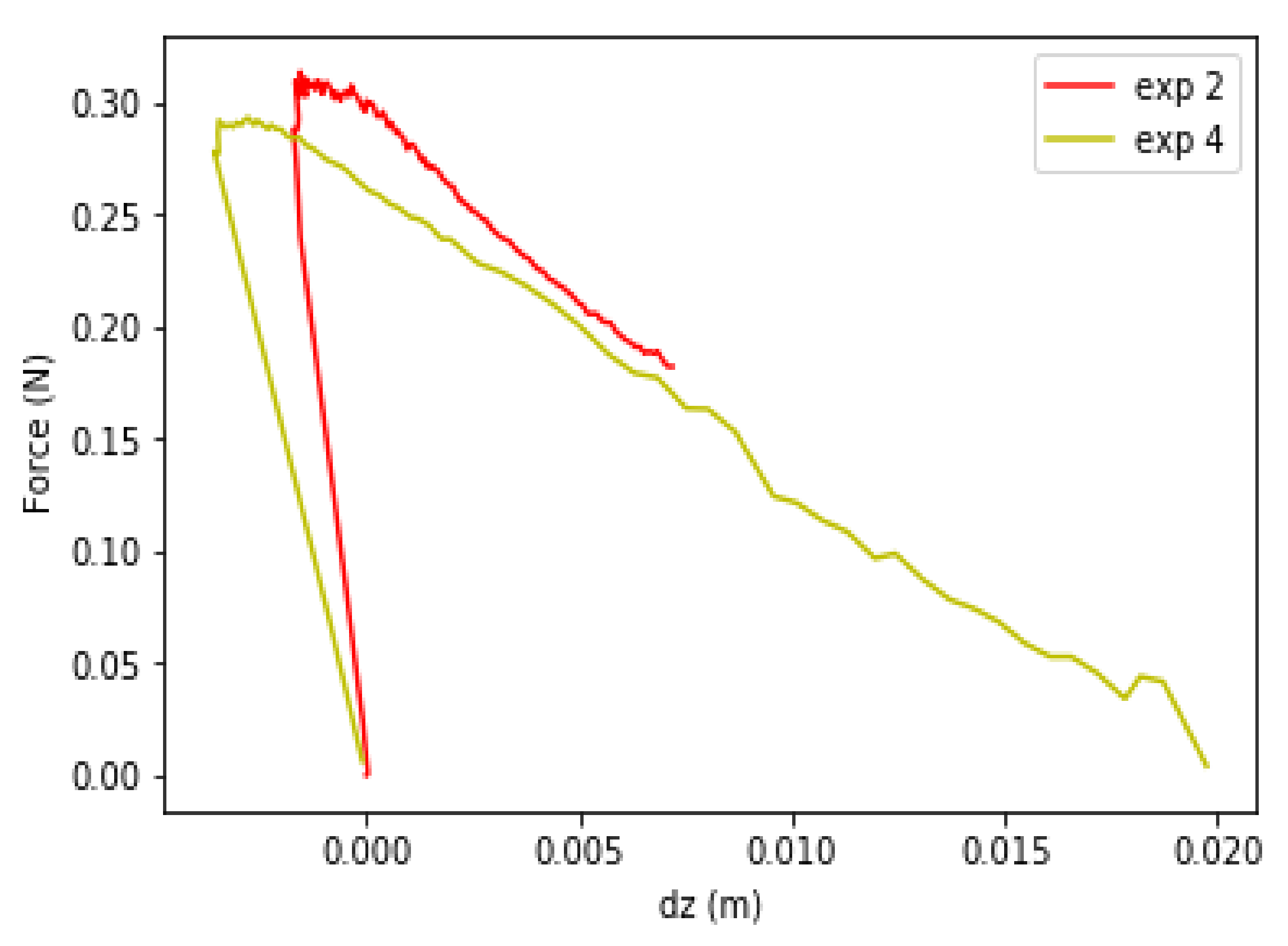
Erreurs systématiques: mesure de la compliance du système





Interprétation : frottement solide

Compliance du système



Prise en compte du décalage dz

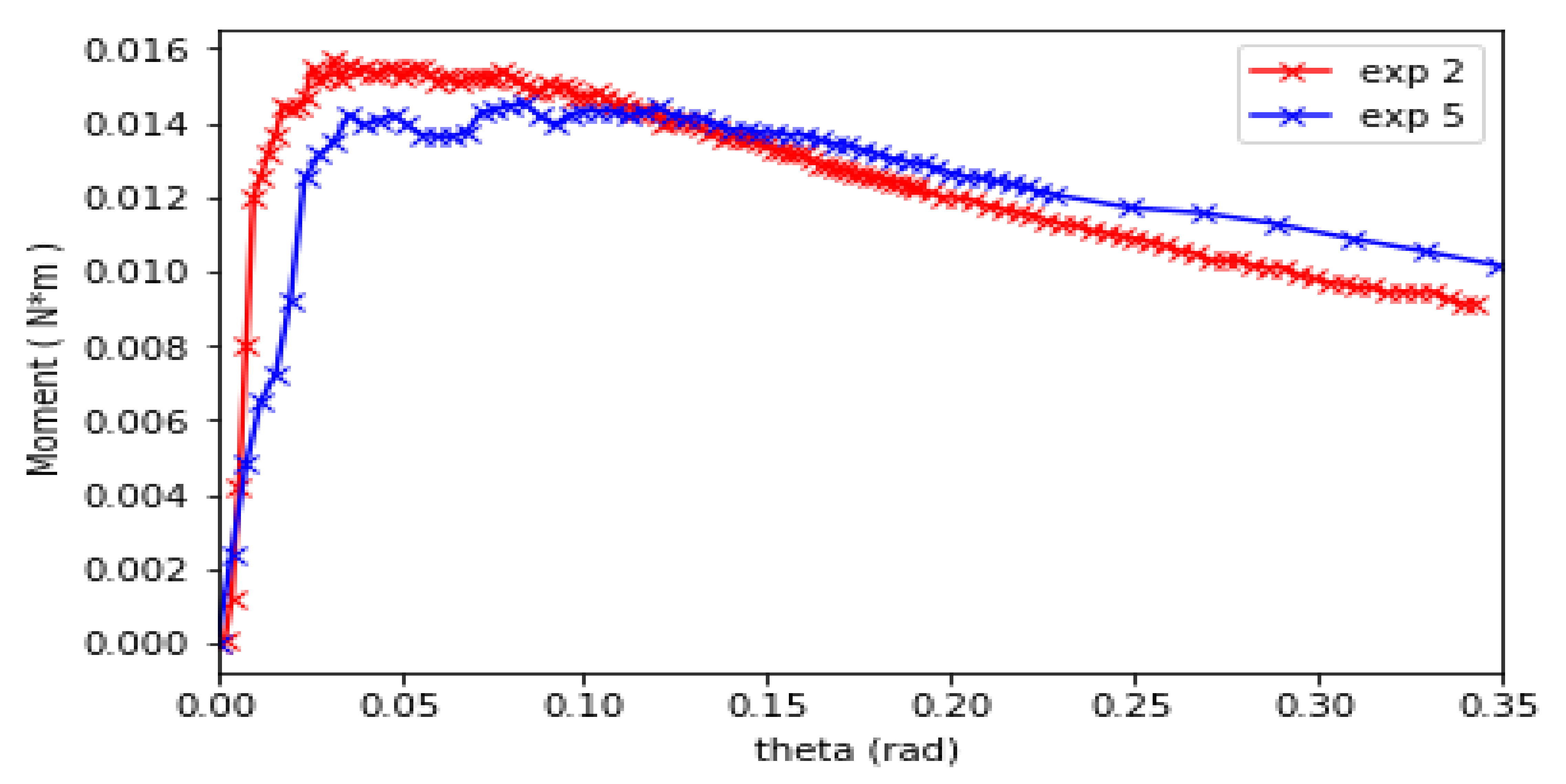


Figure 2: evolution of the Magnetic force moment in fonction of the angle between two magnets (24*4 mm (l*ra) NdFb magnets)

NOUVELLE EXPÉRIENCE PLUS RIGIDE

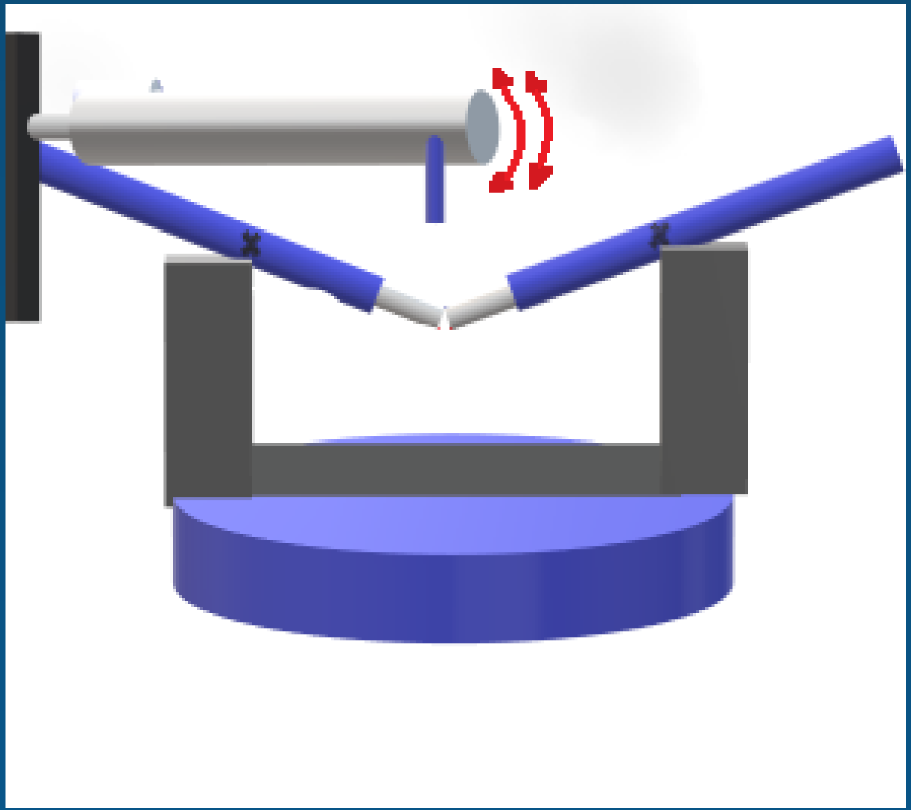


Figure 1: schéma de l'expérience précédente

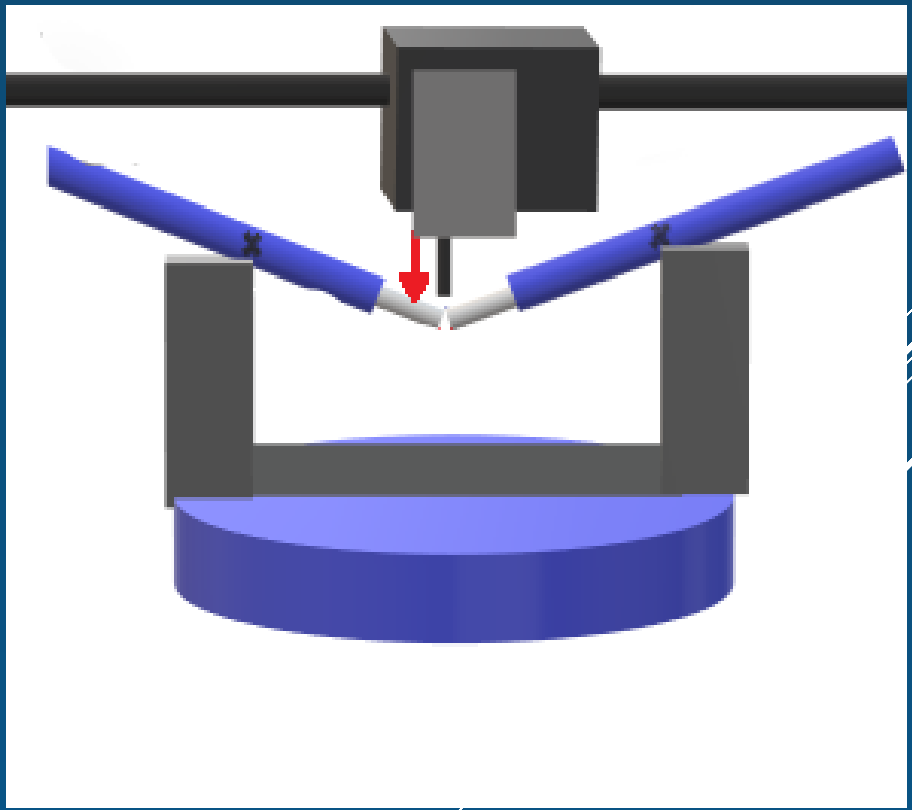
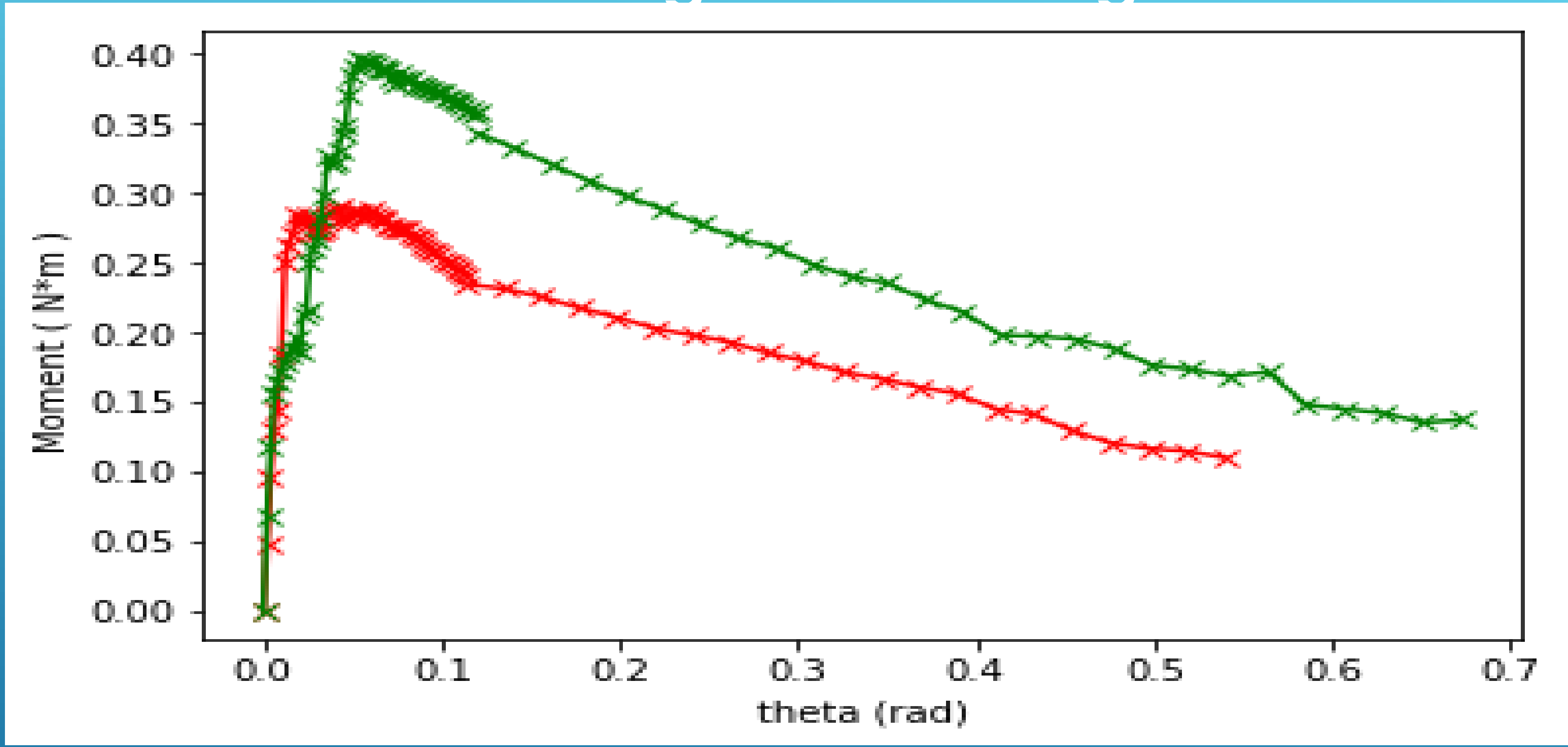


Figure 2: schéma de la nouvelle expérience

Force the circle: Friction on

Variation of the lenght of the magnet



```python
i1= 0.05e-3 #0.05 mm
i2= 0.5e-3 #0.5 mm
I1= 0.1e-3 #0.1 mm
iI= 0.5e-3 #0.5 mm
couleur=['red','blue','green', 'cyan', 'magenta', 'yellow']
def myplot(mes1,mes2,pas1,pas2,clr):

    a1=np.linspace(0,(len(mes1)-1)*pas1,len(mes1))
    a2=np.linspace(a1[-1],a1[-1]+pas2+(len(mes2)-1)*pas2,len(mes2))

    theta5=2*np.arcsin(a1/l)
    theta6=2*np.arcsin(a2/l)

    Mo5 = g *1e-3* np.array(mes1)*l
    Mo6 = g *1e-3* np.array(mes2)*l

    plt.plot(theta5,Mo5,"-x",color= couleur[clr])
    plt.plot(theta6,Mo6,"-x",color= couleur[clr])
    plt.xlabel('theta (rad)')
    plt.ylabel('Moment ( N*m ) ')




myplot(m1, m2, i1, i2,0)
#myplot(M1, M2, i1, i2,2)
#myplot(M3_1, M3_2, i1, i2,4)
myplot(mm1, mm2, i1, i2,5)
myplot(p1, p2, i1, i2,1)
```

PROBLEM

- *A cylindrical chain of magnets possesses a kind of elasticity*
- *Apply a turning moment which curves the chain until it breaks*
- *Can we close the chain on itself so that there is a uniform repartition of the angle between each cylindrical magnet?*
- *If yes, what are the geometrical parametres to consider in order to predict the radius of our cirlce magnet (Rc)?*